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VELOCITY MODIFICATION FOR EARTH CAPTURE OF AN ASTRONOMICAL BODY IN THE SOLAR SYSTEM

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SUMMARY

In studying the possible capture of an astronomical body that is orbiting around the sun, two questions are encountered: How near must it approach the earth before it could be captured? How could we make it approach as near to the earth as required?

This paper answers the first question by using the results of the restricted three-body problem, and partially answers the second question by estimating the order of magnitude of the velocity modification needed to change the orbit of the body and make capture possible.

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VELOCITY MODIFICATION FOR EARTH CAPTURE OF AN ASTRONOMICAL BODY IN THE SOLAR SYSTEM*

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INTRODUCTION

One of the aims of a technological civilization is to conquer nature. Concerning the problems of conquering space we have already many projects such as the landing on the surface of the moon and the probing of the planets. One rather fascinating undertaking in space which has not been widely discussed, perhaps because of its heavy demands on the performance of rockets, is to change the orbit of an asteroid or even of a comet so that is becomes a permanent satellite of the earth.

The reshaping of the solar system requires a large amount of energy. It is evident that we shall not have in the foreseeable future the means for modifying the orbit of the earth in any appreciable degree because its large mass prevents the easy escape of propelling material from its deep potential well. For this reason, what we can modify is only the orbits of those astronomical bodies whose escape velocities are negligible, viz., small asteroids and comets.

In considering the possible capture by the earth of any body that is orbiting around the sun, two questions arise: How near must it approach the earth before it could be captured? How could it be made to approach as near to the earth as required?

PROXIMITY REQUIRED FOR CAPTURE

The first question can be answered by using the results of the restricted three-body problem (Reference 1) which, as has been pointed out before (Reference 2), provide pertinent information concerning the general nature of the motion of a small mass in

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the solar system, especially in the neighborhood of the earth. As an approximation we may regard the motion of a small body in the earth-moon-sun system as two restricted three-body problems. On a smaller scale, the earth and moon provide the two bodies revolving around each other within whose gravitational field the small mass moves. On the larger scale, the earth-plus-moon and the sun form a system of two finite bodies revolving around each other.

In a coordinate system rotating with the two finite bodies around their center of mass, we have, in the standard dimensionless units (Reference 1) for the energy integral of the motion of the third (infinitesimal) body in the restricted three-body problem, the following form

$$V^2 = 2U - C \tag{1}$$

where C is a constant of motion, V is the velocity, and U is a function of the coordinates given by

$$U \equiv \frac{1}{2}(x^2 + y^2) + \frac{1 - \mu}{r_1} + \frac{\mu}{r_2} . \tag{2}$$

Here r_1 and r_2 are the distances of the infinitesimal body from the two finite bodies whose masses are $1-\mu$ and μ respectively, the unit of length being the separation of the two finite bodies.

It follows from Equation 1 that for a given value of C which is determined by the initial conditions alone, the velocity of the infinitesimal body becomes zero on a surface given by

$$2U = C \tag{3}$$

which is therefore known as the zero-velocity surface through which the infinitesimal body cannot penetrate (Reference 1). Thus, corresponding to each value of C there exists a zero-velocity surface. Among the one-parameter family of zero-velocity surfaces there is one which limits the third body indefinitely to the neighborhood of one or the other of the two finite bodies, and which has frequently been called the inner contact surface of the system (Reference 3).

Figure 1 illustrates a part of the inner contact surface for the sun-(earth-plus-moon) system by taking the sun and the center of mass of the earth-plus-moon system as revolving around each other in circular orbits. It shows only the lobe that envelops the earth-moon system, for the lobe enveloping the sun is too large to be drawn to scale.

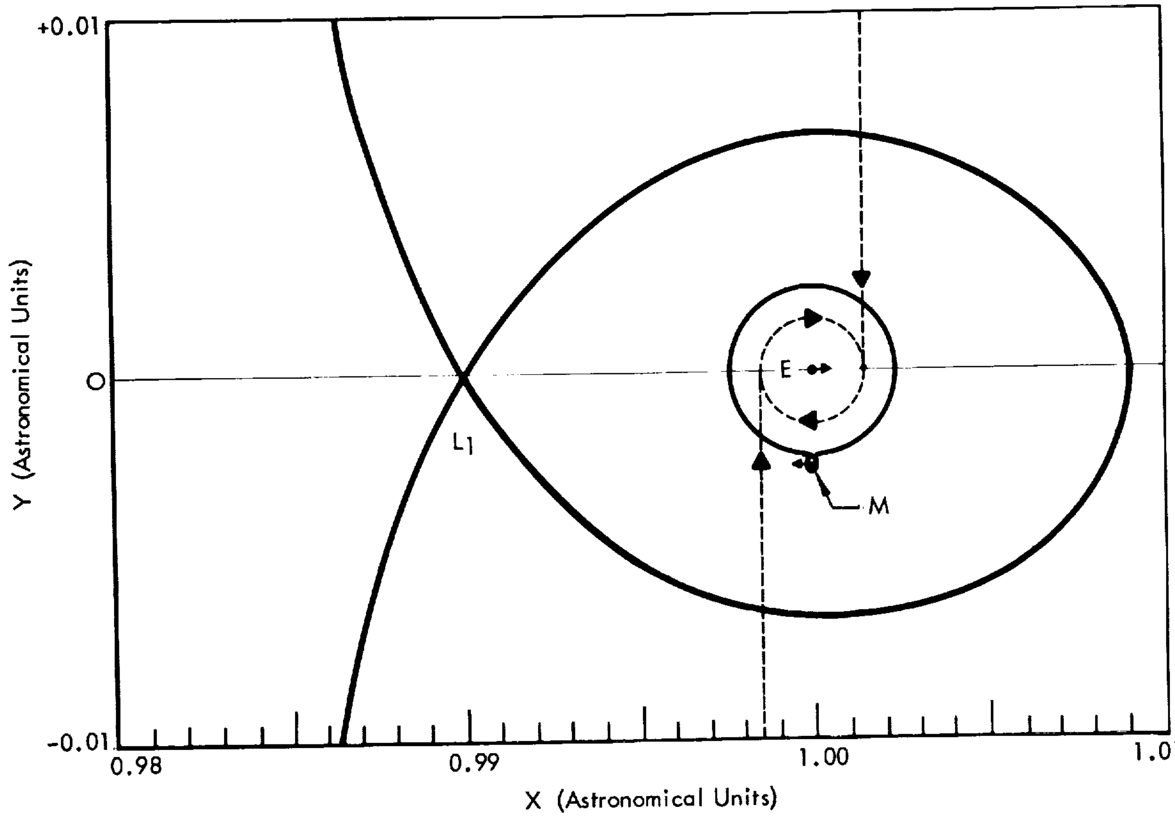

Figure 1 — The inner contact surface, consisting of two lobes – one enveloping the earth, E, and another enveloping the moon, M – of the earth-moon system enclosed in the lobe of the inner contact surface of the sun-(earth-plus-moon) system. This figure illustrates the relative importance of the effects of the sun, the earth, and the moon on the motion of a small body in different regions of space. The dotted lines indicate how asteroids are captured inside the E-lobe.

Thus, with any value of C larger than that corresponding to the inner contact surface, an infintesimal body will move permanently as a satellite of one or the other finite body. If it falls originally within the lobe surrounding the earth-moon, it will remain inside that lobe indefinitely. This, however, does not necessarily mean that it belongs to the earth, because we have thus far treated the earth-moon system only as a single finite body for large-scale motion of the third body.

In order to understand the behavior of the infinitesimal body near the earth-moon system, we must consider another restricted three-body porblem, taking the earth and the moon as two finite bodies revolving around each other in circular orbits. Corresponding to this smaller system we can draw another inner contact surface which is completely shown in Figure 1 and which consists of a lobe surrounding the earth E (called the E-lobe in the following discussion) and a lobe surrounding the moon M. A satellite of the earth has to be inside the E-lobe.

Here we arrive at the answer to our first question: An asteroid has to enter the E-lobe before its capture could be effected. Actually, as has been estimated before (Reference 2), in order to have a satellite revolving around the earth which does not escape in a time-scale of the age of the solar system, the satellite must be located about halfway inside the lobe.

Few known asteroids have ever drifted into the E lobe. Three small asteroids perhaps only a kilometer or two in diameter have been discovered at close approaches to the earth, i.e., at distances of 1.05×10^7 km (Apollo), 2.4×10^6 km (Adonis), and 8×10^5 km (Hermes) (Reference 4), which correspond to 0.070, 0.016, and 0.0054 astronomical unit. The E-lobe, as Figure 1 shows, may be regarded roughly as a sphere of radius 0.0025 astronomical unit. Thus, none of these asteroids has penetrated the E-lobe of the inner contact surface of the earth-moon system. However, Hermes was close to the E-lobe. This face indicates that we can modify the orbits of some small asteroids only slightly in order to bring them into the lobe.

Thus, we may suggest that two steps in changing the velocity of an asteroid must be taken in order to effect its capture. In the first step, which takes place when it is outside the E-lobe, its velocity should be so modified that it will move into the E-lobe. In the second step, which takes place when it is already inside the E-lobe, its velocity should be again modified so that afterwards it will orbit around the earth permanently.

To find the most efficient procedure for capturing a particular asteroid poses a challenging problem to the worker in celestial mechanics; in what follows we attempt only to give an order-of-magnitude estimate of the velocity modification needed for such an undertaking, and thus to determine whether this kind of undertaking is feasible and within our technological reach in the near future (say 25 years).

VELOCITY MODIFICATION REQUIRED FOR PENETRATING THE E-LOBE

Let us first consider the problem of altering the orbit of an asteroid such that it could enter the E-lobe sometime later. Assume the original orbit of the asteroid to be circular with radius a_o. We shall consider only two special cases in order to see the order of magnitude of the required change in velocity. Case 1: If the radius of the orbit of the asteroid is originally greater than that of the earth, that is, $a_o > 1$,

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we consider the special case that after its velocity has been modified, the asteroid will enter the E-lobe at its perihelion. Case 2: If $a_0 < 1$, we consider the special case that the asteroid will enter the E-lobe at its aphelion.

Denoting by a_1 and e_1 the semi-major axis and the eccentricity of the altered orbit, we must have

$$a_1(1 - e_1) = 1 \tag{4}$$

in order that it could be captured at its perihelion by the earth. Choose a polar coordinate system with the sun at the pole. If the velocity modification is affected at $A(a_0, \theta)$ as in Figure 2a,

$$e_1 = \frac{a_0 - 1}{1 + a_0 \cos\theta}. \tag{5}$$

Equations 4 and 5 determine a_1 and e_1 when a_0 is given. It is now an easy matter to compute the difference in velocity of the asteroid on the circuit orbit (a_0) and on the elliptical orbit (a_1, e_1) at Point A. The calculation is simple and straightforward with the following result for the differences in velocity components:

$$\Delta V_r = \frac{1 - a_0}{a_0^{1/2}} \left(\frac{1 - \cos\theta}{1 + a_0 \cos\theta} \right)^{1/2} \tag{6}$$

and

$$\Delta V_\theta = \frac{1}{a_0^{1/2}} \left[\left(\frac{1 + \cos\theta}{1 + a_0 \cos\theta} \right)^{1/2} - 1 \right]. \tag{7}$$

In the second special case (Figure 2b), we consider capture at the aphelion of its modified orbit in the event $a_0 < 1$. Thus,

$$a_1(1 + e_1) = 1. \tag{8}$$

The eccentricity of the modified orbit is then given by

$$e_1 = \frac{1 - a_0}{1 - a_0 \cos\theta}, \tag{9}$$

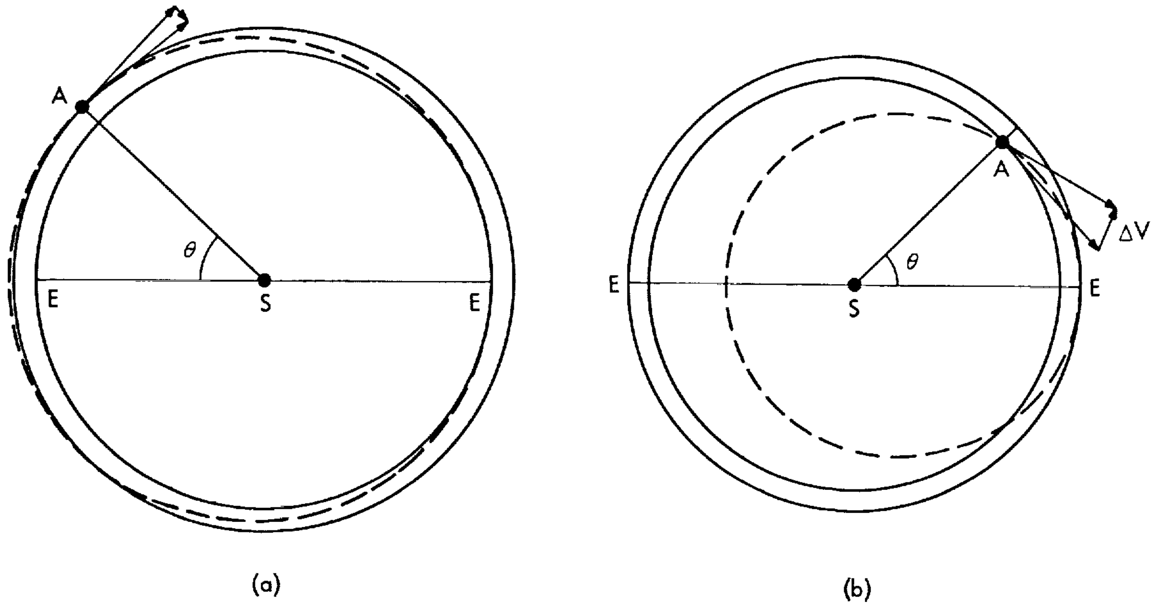

Figure 2 — Two special cases of modifying the velocity of an asteroid so that its modified orbit touches the orbit of the earth. The two circles in each diagram represent the orbit of the earth (EE) and the original orbit of the asteroid which is, in (a), outside the earth's orbit and, in (b), inside the earth's orbit. S is the sun and A is the point at which the velocity modification (ΔV) is instantaneously applied. The ellipse shown by the broken line is the modified orbit.

and the differences in velocity components at A of the asteroid on the circular orbit (a_0) and on the elliptical orbit (a_1, e_1) are given by

$$\Delta V_r = \frac{1 - a_0}{a_0^{1/2}} \left(\frac{1 + \cos\theta}{1 - a_0 \cos\theta} \right)^{1/2} \tag{10}$$

and

$$\Delta V_\theta = \frac{1}{a_0^{1/2}} \left[\left(\frac{1 - \cos\theta}{1 - a_0 \cos\theta} \right)^{1/2} - 1 \right] \tag{11}$$

The difference in velocity calculated in this way obviously represents the velocity correction that has to be applied in order to bring the asteroid from a circular orbit into an elliptical orbit that touches the orbit of the earth.

Table 1 gives some numerical results derived from Equations 6-7 and 10-11 for different values of θ. From this table we can immediately see that in both cases the most advantageous point at which the modification of velocity can be applied is the

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Table 1

The Eccentricity e_1 of the Modified Orbit and the Velocity Changes Required to Bring the Asteroid into the E-Lobe

θ (degrees)	$a_0 = 1.05$ e_1	ΔV_r (km/sec)	ΔV_θ (km/sec)	$a_0 = 0.95$ e_1	ΔV_r (km/sec)	ΔV_θ (km/sec)
0	0.024	0	-0.37	1	9.6	-31
30	0.026	-0.38	-0.34	0.28	5.0	- 4.2
45	0.029	-0.60	-0.31	0.15	3.5	- 1.6
60	0.033	-0.84	-0.23	0.09	2.6	-0.73
90	0.050	-1.5	0	0.05	1.5	0
120	0.11	-2.6	+ .75	0.034	0.89	+0.27
135	0.19	-3.7	+1.9	0.030	0.64	+0.31
150	0.55	-6.6	+6.2	0.027	0.41	+0.37
180	-	-	-	0.026	0	+0.40

point where the asteroid is as far as possible from the point where it will actually enter the E-lobe after velocity modification. In both the cases considered here, the optimum velocity change amounts only to a fraction of one km/sec. Intuitively this conclusion is self-evident because the orbit is least changed under this circumstance. Indeed, it is familiar when we reflect upon the most efficient way, energywise, of sending a rocket to Venus or Mars from the earth.

VELOCITY MODIFICATION REQUIRED FOR CAPTURE

Assume that an asteroid has already been moving along an orbit (a_1, e_1) which lies in the same plane as the orbit of the earth and which could penetrate into the E-lobe of the inner contact surface. Let us now investigate the modification velocity required to keep it permanently inside the lobe. We can easily find its velocity at any point on the orbit in the frame of reference rotating with the earth-moon system around the sun, and consequently determine the value of C in Equation 1. When the asteroid is far away from the earth-moon system, we can compute the value of C by neglecting μ. Thus,

$$C = \frac{1}{a_1} + 2\left(1 - e_1^2\right)^{1/2} a_1^{1/2}. \tag{12}$$

Since C is a constant of motion in the restricted three-body problem, we can derive the velocity of the asteroid from Equations 1, 2, and 12 when it reaches a point inside the E-lobe. Then μ in the term $2\mu/r_2$ of Equation 2 can no longer be neglected, although we may still set $\mu=0$ in the term $2(1-\mu)/r_1$. Also, we may write

$$r_1^2 = x^2 + y^2$$

to a better approximation. It is obvious that r_1 must be very near to unity. Consequently the velocity V of the asteroid when it is inside the E-lobe is given by

$$V_1^2 = r_1^2 + \frac{2}{r_1} + \frac{2\mu}{r_2} - \left[\frac{1}{a_1} + 2(1-e_1^2)^{1/2} a_1^{1/2}\right] , \tag{13}$$

the effect of the moon being neglected in this order-of-magnitude calculation.

We shall consider only two special situations of capture, namely, that the earth and the asteroid have the closest encounter (1) at the aphelion of the asteroid if $a_1 < 1$, or (2) at its perihelion if $a_1 > 1$; and that its velocity is modified at this point of the closest encounter. From Equation 13 it follows that, for the first case,

$$V_1^2 = \left[\left(\frac{1-e_1}{r_c}\right)^{1/2} - r_c\right]^2 + \frac{2\mu}{|1-r_c|} , \tag{14}$$

while for the second case,

$$V_1^2 = \left[\left(\frac{1+e_1}{r_c}\right)^{1/2} - r_c\right]^2 + \frac{2\mu}{|1-r_c|} , \tag{15}$$

where r_c is the value of r_1 at the point of velocity modification. Thus,

$$r_c = a_1(1+e_1) \tag{16}$$

for the first instance and

$$r_c = a_1(1-e_1) \tag{17}$$

for the second. The last term in both Equations 14 and 15 represents the increase in kinetic energy when the asteroid falls into the E-lobe as a result of the earth's attraction.

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On the other hand, the orbital velocity of a satellite around the earth in the frame of reference we refer to is given by

$$V_0 = \frac{\alpha}{|1 - r_c|} \tag{18}$$

where α is the ratio of the mass of the earth to that of the sun. Therefore, the required change in velocity ΔV (inside the E-lobe) in order to capture the asteroid is between $|V_1 - V_0|$ and $|V_1 + V_0|$. The actual value of ΔV depends upon the direction of the velocity vectors, but apparently lies in most cases near the lower limit. The computed values of $V_1 - V_0$ and $V_1 + V_0$ in both cases are given in Table 2 for two different values of $|1 - r_c|$ which measures the size of the orbit around the earth after the asteroid has been captured. The case of $|1 - r_c| = 0.00028$ corresponds to making an asteroid an earth satellite with a period of 24 hours, while the case $|1 - r_c| = 0.002$ corresponds to an orbit near

would then move along the same orbit as that of the earth and consequently the two would remain at a constant distance apart. Needless to say, the unit of velocity used here is the orbital velocity of the earth around the sun, but we have converted it into km/sec in Tables 1 and 2 from which it is evident that the required change is not large.

Thus, we can capture the hypothetical asteroids (a_0 = 0.95, 1.05) considered here by using a rocket which is able to change the asteroid's velocity by only one or two km/sec in optimum circumstances. This value can be compared with 11 km/sec for the escape velocity from the earth and is indeed small astronomically. But the mass of even a small (one kilometer in radius) asteroid is of the order of 10^{16} grams, and the rocket used to modify its velocity has to be lifted from the earth, although conceivably we may be able to use material on the asteroid as ejecting matter. This proposed project would require a major effort but, if realized, it would be a lasting mark of human achievement.

CONCLUDING REMARKS

We have illustrated by a few hypothetical examples that the necessary changes in velocity for capturing an asteroid could be small. When time comes for actually carrying out the capture, the modification of velocity may not be applied at two points in a short time interval as is assumed here. Instead, a low thrust applied for a long time will most likely be used for steering an asteroid into an orbit around the earth. The present examples only emphasize the feasibility of this kind of undertaking in view of the small change in velocity necessary for its success. Therefore, the immediate problem is to search for small asteroids that come naturally to within 0.05 astronomical unit or r_0 of the orbit of the earth. After its orbit is determined, an extensive calculation should then be carried out on a high-speed digital computer in order to determine the most efficient way of capturing it. This will be a very interesting problem in celestial mechanics.

If we put an asteroid one kilometer in radius into the 24-hour orbit, its apparent photovisual magnitude in the full phase (when not in eclipse) will be

$$m = -2.23 - 2.5 \log A \tag{19}$$

where A is its albedo in the photovisual region and its angular diameter will be about 10 seconds of arc. Taking A = 0.1, which is of the order of magnitude of the albedo of the moon, we have $m \approx 0.27$ mag. It would be as bright as the brightest stars in the sky. Actually, because of the irregular shape and the rotation of the asteroid, the brightness will fluctuate around this value.

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